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$\mathbb{R}^n$  is a vector space over  $\mathbb{R}$ . The set of all linear transformations from  $\mathbb{R}^n$  to  $\mathbb{R}^n$  is denoted by  $\mathcal{L}(\mathbb{R}^n, \mathbb{R}^n)$ . This set is a vector space over  $\mathbb{R}$  with addition and scalar multiplication defined as follows:
 
$$(A+B)(x) = Ax + Bx$$

$$(cA)(x) = c(Ax)$$
 for  $A, B \in \mathcal{L}(\mathbb{R}^n, \mathbb{R}^n)$  and  $c \in \mathbb{R}$ . The zero element is the zero transformation  $0$ .

Let  $A \in \mathcal{L}(\mathbb{R}^n, \mathbb{R}^n)$ . The image of  $A$  is the set  $\text{Im } A = \{Ax \mid x \in \mathbb{R}^n\}$ . The kernel of  $A$  is the set  $\text{Ker } A = \{x \in \mathbb{R}^n \mid Ax = 0\}$ . The rank of  $A$  is the dimension of  $\text{Im } A$ , and the nullity of  $A$  is the dimension of  $\text{Ker } A$ . The Rank-Nullity Theorem states that
 
$$\text{rank } A + \text{nullity } A = n$$
 for any  $A \in \mathcal{L}(\mathbb{R}^n, \mathbb{R}^n)$ .

Let  $A \in \mathcal{L}(\mathbb{R}^n, \mathbb{R}^n)$ . The matrix of  $A$  with respect to the standard basis of  $\mathbb{R}^n$  is denoted by  $[A]$ . The matrix  $[A]$  is an  $n \times n$  matrix with entries in  $\mathbb{R}$ . The matrix  $[A]$  is invertible if and only if  $\text{Ker } A = \{0\}$ , which is equivalent to  $\text{rank } A = n$ . In this case, the inverse matrix  $[A^{-1}]$  is the matrix of the inverse transformation  $A^{-1}$ .





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