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\mathbb{R}^n is a vector space over \mathbb{R} . The set of all linear transformations from \mathbb{R}^n to \mathbb{R}^n is denoted by $\mathcal{L}(\mathbb{R}^n, \mathbb{R}^n)$. This set is a vector space over \mathbb{R} with addition and scalar multiplication defined as follows:

$$(T + S)(v) = T(v) + S(v)$$

$$(cT)(v) = cT(v)$$
 where $T, S \in \mathcal{L}(\mathbb{R}^n, \mathbb{R}^n)$ and $c \in \mathbb{R}$. The zero element is the zero transformation 0 .

Let $T \in \mathcal{L}(\mathbb{R}^n, \mathbb{R}^n)$. The kernel of T , denoted by $\ker T$, is the set of all vectors $v \in \mathbb{R}^n$ such that $T(v) = 0$. The image of T , denoted by $\text{Im } T$, is the set of all vectors $w \in \mathbb{R}^n$ such that $w = T(v)$ for some $v \in \mathbb{R}^n$. The rank-nullity theorem states that

$$\dim \ker T + \dim \text{Im } T = n$$
 where $n = \dim \mathbb{R}^n$.

Let $T \in \mathcal{L}(\mathbb{R}^n, \mathbb{R}^n)$. The matrix of T with respect to the standard basis $\{e_1, \dots, e_n\}$ is denoted by $[T]$. The matrix $[T]$ is an $n \times n$ matrix with entries in \mathbb{R} . The matrix $[T]$ is invertible if and only if T is an isomorphism. The determinant of $[T]$ is denoted by $\det [T]$. The trace of $[T]$ is denoted by $\text{tr } [T]$.

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